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RELATION BETWEEN IQHE AND FQHE WITH BERRY PHASE

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Abstract: It is shown here that a particle in an intense magnetic field may acquire the Berry phase and the topological features associated with this phase may be taken to be responsible for both IQHE and FQHE. The two different manifestations of quantum Hall effect has been realized in a unified scheme where the electrons associated with fractional quantum Hall effect are found to be in excited state having higher angular momentum.

Keywords: Quantum Hall Effect, Berry phase.

Quantum phenomenon [1] in Hall effect is obsvered when the condition $\hbar\omega_c > k_B T$ is achieved for which the transverse conductivity or Hall conductivity, $\sigma_H = \sigma_{xy} = \nu e^2/h$ (where ν = integer or fraction) remains constant over finite intervals-plateaus with the variation of magnetic field or carrier density.The IQHE for integer ν is believed to be a manifestation of the non-interacting electrons in presence of impurity in the sample.On the other hand FQHE for fractional ν is believed to arise from a condensation of the two dimensional(2D) electrons into a new collective state of matter as a result of repulsive inter-electron interactions.

The fundamental difference between IQHE and FQHE lies in the fact that the latter is observed in extremly clean sample in contrast to the situation when impurity is found to be an essential ingredient for pronounced integrally quantized Hall effect. This essentially differtent feature prompted many authors to suggest that the two types of quantum Hall effect might nothing common in these.

Jain[2] first proposed that the two different maifestation of IQHE and FQHE can be dealt in unified scheme with same underlying physics. He posed the fractational quantum Hall effect of electrons as manifestation of the integer quantum Hall effect of composite fermionic objects consisting of electrons bound to flux quantum.

In this paper we want to point out that the external intense magnetic field causes chiral symmtry breaking of electrons (Hall particles) and as a result, anomaly is realized through Berry phase[3] in association with the quantization of Hall conductivity,when topological features associated with space-time properties are taken into account. Indeed, we here pointed out that a common origin between both IQHE and FQHE can be drawn from angular momentum aspect where particles associated with fractional quantum Hall effect are found to be in excited state having higher angular momentum.

Here we followed the technical innovation described by Haldane[4] where he considered 2D electron gas of N particles on the spherical surface of radius R, in a radial (monopole) magnetic field $B=\hbar S/eR^2$ (>0) where 2S is an integer. The dynamical angular momentum followed by a single particle is

$$\bar{\Lambda} = \bar{r} \times [\ -i\hbar\nabla + e\bar{A}(\hat{r})\ ] \tag{1}$$

This monopole type behavior can be realized through the anisotropic feature of local three dimensional space where the momentum coordinate do not commute[5].

$$[\ p_i, p_j\ ] = i\mu\epsilon^{ijk} \frac{x^k}{r^3} \tag{2}$$

Infact, the motion of a charged particle in the field of magnetic monopole, is equivalent to the motion of the particle in the anisotropic space where the conserved angular momentum is given by

$$\bar{J} = \bar{r}\times\bar{p} - \mu\bar{r} \tag{3}$$

Here μ is the measure of anisotropy which behaves like the strength of magnetic monopole.

In analogy between the quantization of Hall particle with the spin polarization, we can describe this spin system by a direction vector ξ^μ attached to a space-time point x^μ when the two opposite orientation of ξ^μ give rise to two opposite helities corresponding to spin up and spin down states. Infact the coordinate can be denoted as[6]

$$z^{\mu} = x^\mu + i\xi^\mu = x^\mu + \frac{i}{2}\lambda^\mu_\alpha\, \theta^\alpha \qquad (\alpha=1,2) \tag{4}$$

Here ξ^μ is associated with a two component spinorial variable θ. The complex conjugate of this chiral coordinate will give rise to the opposite helicity The wave-function $\phi(z^{\mu})=\phi(x^{\mu}+i\xi^{\mu})$ of the state should take into account an extra polar coordinate $\varkappa$ along with r,θ and ϕ. Here $\varkappa$ specifies the rotational orientation around the direction vector. It has been found that this wave function $\phi(z^{\mu})$ is best described by the spherical harmonics $Y_l^{m,\mu}$ studied by Fierz[7] and Hurst[8]. Following them we can write

$$
\begin{aligned}
Y_{1/2}^{\frac{1}{2},\frac{1}{2}} &= \sin(\theta/2)e^{i(\phi-\varkappa)/2} \\
Y^{-\frac{1}{2},\frac{1}{2}} &= \cos(\theta/2)e^{-i(\phi+\varkappa)/2} \\
Y^{\frac{1}{2},-\frac{1}{2}} &= \cos(\theta/2)e^{i(\phi+\varkappa)/2} \\
Y_{1/2}^{-\frac{1}{2},-\frac{1}{2}} &= \sin(\theta/2)e^{-i(\phi-\varkappa)/2}
\end{aligned}
\tag{5}
$$

by which we can reformulate Haldane's wave function for N-particle as follows

$$\psi_N^m = \prod_{i>j}(u_i v_j - u_j v_i)^m \tag{6}$$

where $m=1/\nu$ and $\theta = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} Y_{1/2}^{1/2,1/2} \\ Y_{1/2}^{-1/2,1/2} \end{bmatrix}$ a two component spinor.

Evidently to follow Fermi statistics m should be odd m=1,3,5.... Since m is an integer,we can identify, following Haldane, as $m = J_{ij} = J_i + J_j$ for N = 3 particles. The lowest level angular momentum is J = 1/2 due to $|\mu|$ = 1 = 1/2 and belongs to ground state for $\bar{r}\kappa\bar{p}$ = o which leads m = 1 or ν = 1 IQHE states.

We see that following Dirac quantization condition $e\mu$ = 1/2 this ν = 1 states have fermion number 1.

However, if we consider the next excited state with $\bar{r}\times\bar{p}$ = 1 the respective anguler momentum is changed to J = 3/2 for μ = 1/2. This can be viewed as a system with μ_{eff} = 3/2 having $\bar{r}\times\bar{p}$ = o. Hence for three particles state (N = 3) where each electron carries 3/2 angular momentum the filling factor is fractional for ν = 1/m = 1/3 where m = 3/2 + 3/2 = 3. In this excited state the fermion number of the quasiparticle is 1/3.

In this way, the 1/5 filling factor of FQHE states corresponds to the excited state having $\bar{r}\times\bar{p}$ = 2. Thus we see that with the increase of angular momentum the ground state at ν = 1 becomes excited at ν = 1/3. Other FQHE states are obtained from the parent state with the effect of majority spin.

It has been pointed out by many authers that the phenomenon of Quantum Hall effect is caused by nontrivial topological transformation. On the surface of 3D sphere the Hall will have the topological action[9]

$$W_\theta = \frac{\theta}{16\pi^2}\int \mathrm{Tr}\, {}^*F_{\mu\nu}F_{\mu\nu}\, d^3x \quad \text{where } \theta = \frac{\sigma_H}{c^2} \tag{7}$$

The topological term ${}^*F_{\mu\nu}F_{\mu\nu}$ which gives rise to chiral anomaly, is a total divergence $\partial_\mu\Omega_\mu$ (where Ω_μ is the Chern–Simon secondary characteristic class) that measures the topological index[10] $q=\int\partial_\mu\Omega_\mu d^4x$. The parallel transport of the quantized Hall particle over a closed path gives the Berry's topological phase $e^{2\pi\mu i\theta}$ which is related to q through $q = 2\mu$[11]. Considering $m = 2\mu$ for μ being half integer we have the required phase as

$$\phi_B = m\pi\theta = (1/\nu)\pi(\sigma_H/c^2) = (1/\nu)\pi(\nu e^2/hc^2) \tag{8}$$

Though eventually Berry phase seems to be independent of filling factor ν, yet it changes with the change of e which varies from e to e/s as ν changes to 1/3 from 1. We thus conclude that the Berry phase will change from one plateau region to another owing to the change of effective fermion number of quasiparticles. It can bee added that the phase factor in plateaus for higher filling factor $\nu > 1/5$ perhaps cannot be visible due to crystallization in the Hall states[12].

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